The prime symbol denotes exclusion from the sum of the quantities having the same subscripts $\nu$ and $n$. The downwash factor $\omega$ is defined by the relation

$$\omega = 2n \tag{10}$$

Reference 1 takes an average value of $\omega$ equal to $2n_s$, which is the sheared value for a swept wing $[\lambda(y) = 0]$. For a 45° swept wing with aspect ratios of 3, 5, and 7, the value for $\omega$ is 1.025, 1.010, and 1.005, respectively. Strictly speaking, $\omega$ is also dependent on the spanwise location and would vary in the same manner as $n$ varies, since $\omega = 2n$.

The load distribution obtained by using Eq. (9), for $m = 31$, is shown in Figs. 2a, 2b, 2c, and 2d for the wing planforms of Refs. 4, 5, 6, and 7, respectively. In accordance with Refs. 1 and 10, the value for $\omega$ was held constant; a value of unity was assigned following Ref. 10. Figure 2 shows the Kuchemann-Multhopp lifting line method using aerodynamic centers obtained by the methods of Refs. 1, 2, and 8 on the upper set of curves in each figure. The difference in load distribution between that calculated using the Kuchemann tangent approximation and the Kuchemann hyperbolic approximation is small, so that the loading curves shown make use of the tangent approximation. In addition, no consideration was given to wing thickness effects, except in its effect on the aerodynamic center position in using the method of Ref. 2, in the theoretical calculations so the value of $a_0$ used was $2\pi$. In general, of the loadings obtained by the Kuchemann-Multhopp lifting line method, that obtained from using the aerodynamic centers from the Kuchemann tangent approximation appears to give the best agreement with experiment except for the case of wing D. The loading for wing D appears to be best represented by use of either the aerodynamic centers from the Multhopp lifting surface theory[8] or the Transonic Data Memorandum method[2] without the thickness correction. The dotted curves shown in Fig. 2c show the slight reduction in the calculated loading obtained by using the extended aerodynamic center distribution shown by the dashed lines in Fig. 1c.

Figure 2 also shows the span loading characteristics of wings A, B, C, and D as obtained from the Weissinger[11] (using 59 spanwise points) and Multhopp lifting surface[8] (using 4 chordwise and 31 spanwise points) theories compared with experiment. This figure shows that the loadings predicted by the Weissinger theory and Multhopp lifting surface theory give comparable results and agree quite well with experiment except in the case of wing D where both theories slightly underpredict the level of the loading on the wing.

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# Change in Pitching-Moment Coefficient Due to Ground Effect

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### Nomenclature

$a_t$ = slope of tail lift coefficient
$C_L$ = lift coefficient
$C_m$ = pitching-moment coefficient
$\bar{c}$ = mean aerodynamic chord
$q$ = dynamic pressure
$S$ = wing area
$\alpha$ = angle of attack
$\Delta$ = denoting the change due to ground effect in case of constant angle of attack, e.g., $\Delta C_L = C_L - C_{L_0}$
$\delta_e$ = elevator angle
$\epsilon$ = downwash angle at the tail
$\tau$ = relative control effectiveness, $\tau = \partial\alpha_t/\partial\delta_e$

***Subscripts***

$t$ = tail
$w$ = wing-body combination
$0$ = free air, e.g., $C_{L_0}$
$|_\delta$ = constant elevator angle

REFERRING to a new flight-test method for measurement of ground effect of fixed wing aircraft,[1] this Note is concerned with the change in pitching-moment coefficient due to ground effect and the factors causing this change in case of a constant-angle-of-attack approach. The investigation described herein has been derived from a method which was used for evaluating the change in downwash angle at the tail when measuring ground effect of the Transall C-160 airplane.[2]

Dividing up the forces and moments as shown in Fig. 1, the change in lift coefficient due to ground effect can be expressed as

$$\Delta C_L = \Delta C_{L_w} + k_t(\Delta C_{L_t}|_\delta + \tau a_t \Delta\delta_e) \tag{1}$$

where terms of small magnitude are neglected and

$$k_t = S_t q_{t_0}/S q_0 = S_t(q_{t_0} + \Delta q_t)/[S(q_0 + \Delta q)] \tag{2}$$

is assumed to be constant. The difference between the moment equations in free air and in ground proximity yields

$$x_a\, \Delta C_{L_w} - k_t(l'_t - x_a)(\Delta C_{L_t}|_\delta + \tau a_t \Delta\delta_e) = 0 \tag{3}$$

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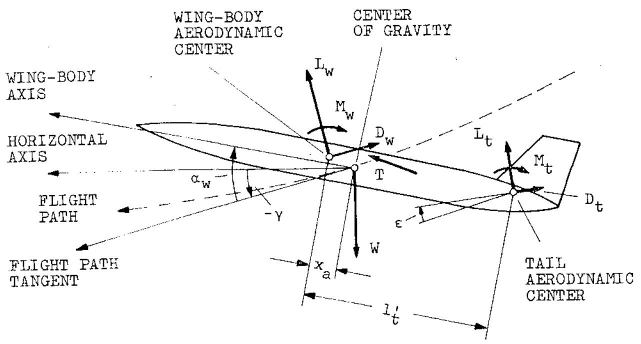


**Fig. 1 Forces and moments.**

Here terms of small order are neglected too, such as the differences in the effects of drag as well as thrust on pitching moment, acceleration and damping effects or the change in wing-body pitching-moment coefficient at $C_{Lw} = 0$. In consequence of appropriate choice of $\alpha_w$ (i.e., the wing-body reference axis) $x_a$ the effective lever arm of $\Delta C_{Lw}$.

The change in pitching-moment coefficient due to ground effect in case of constant elevator angle ($\Delta\delta_e = 0$) is, from (3),

$$\Delta C_m = \frac{x_a}{\bar{c}}\,\Delta C_{Lw} - k_t\,\frac{l'_t}{\bar{c}}\left(1 - \frac{x_a}{l'_t}\right)\Delta C_{Lt}|_\delta \qquad (4a)$$

or, using the measured values of the change in elevator angle,

$$\Delta C_m = k_t\,\frac{l'_t}{\bar{c}}\left(1 - \frac{x_a}{l'_t}\right)(a_{t0} + \Delta a_t)\Delta\delta_e \qquad (4b)$$

The change in tail lift coefficient in case of constant elevator angle is given by

$$\Delta C_{Lt}|_\delta = \Delta a_t(\alpha_t - \epsilon_0 + \tau\delta_{e0}) - (a_{t0} + \Delta a_t)\Delta\epsilon \qquad (5)$$

The initial steady-state conditions are

$$C_{Lw_0} \doteqdot C_{L0} - k_t C_{Lt_0} \qquad (6)$$

$$C_{Lt_0} = a_{t0}(\alpha_t - \epsilon_0 + \tau\delta_{e0}) \qquad (7)$$

and

$$C_{m0_w}{}^* + \frac{x_a}{\bar{c}}\,C_{Lw_0} - k_t\,\frac{l'_t}{\bar{c}}\left(1 - \frac{x_a}{l'_t}\right)C_{Lt_0} = 0 \qquad (8)$$

where $C_{m0_w}{}^*$ includes effects of drag and thrust on pitching moment.

With the use of these equations and on the assumption that $\tau$ is constant, the change in pitching-moment coefficient is found to be

$$\Delta C_m = \frac{x_a}{\bar{c}}\,C_{Lw_0}\left[\frac{\Delta C_{Lw}}{C_{Lw_0}} - \frac{\Delta a_t}{a_{t0}} - k_t a_{t0}\left(1 + \frac{\Delta a_t}{a_{t0}}\right)\times \frac{\Delta\epsilon}{C_{Lw_0}}\right] + k_t\,\frac{l'_t}{\bar{c}}\,a_{t0}\left(1 + \frac{\Delta a_t}{a_{t0}}\right)\Delta\epsilon - \frac{\Delta a_t}{a_{t0}}\,C_{m0_w}{}^* \qquad (9a)$$

or, since from (1) and (3)

$$\Delta C_{Lw} = \left(1 - \frac{x_a}{l'_t}\right)\Delta C_L$$

it can be expressed as

$$\Delta C_m = \left(1 - \frac{x_a}{l'_t}\right)\left\{C_{L0}\left[\frac{x_a}{\bar{c}}\left(\frac{\Delta C_L}{C_{L0}} - \frac{\Delta a_t}{a_{t0}}\right) + k_t\,\frac{l'_t}{\bar{c}}\,a_{t0}\left(1 + \frac{\Delta a_t}{a_{t0}}\right)\frac{\Delta\epsilon}{C_{L0}}\right] - \frac{\Delta a_t}{a_{t0}}\,C_{m0_w}{}^*\right\} \qquad (9b)$$

An approximation can be derived from (9b), if $\Delta a_t/a_{t0} \ll \Delta C_L/C_{L0}$ (i.e., the tail is not too close to the ground in comparison with the wing) and $C_{m0_w}{}^*$ is not too large compared with $C_{L0}$. As in addition $x_a/l'_t$ is usually small ($x_a/l'_t \ll 1$), it follows that

$$\Delta C_m \doteqdot C_{L0}\left(\frac{x_a}{\bar{c}}\,\frac{\Delta C_L}{C_{L0}} + k_t\,\frac{l'_t}{\bar{c}}\,a_{t0}\,\frac{\Delta\epsilon}{C_{L0}}\right) \qquad (10)$$

Examination of (9) and (10) shows the main factors causing a change in pitching-moment coefficient due to ground effect: the reduction in downwash angle at the tail $\Delta\epsilon$, and the increase in lift of wing-body combination $\Delta C_{Lw}$ and in tail lift slope $\Delta a_t$ combined with the position of center of gravity $x_a$.

Usually, when $\Delta C_{Lw}$ and $\Delta\epsilon$ are the dominant parameters, the most negative change in pitching-moment coefficient occurs for the most forward position of center of gravity because the factor of $x_a/\bar{c}$ in (9a) and (10) is positive.

Being sufficiently large, the terms containing $x_a$ may equalize the remaining ones because their effects on $\Delta C_m$ are opposite in case of different signs. Thus, it is possible, that the change in pitching-moment coefficient is negligible and a change in elevator angle is not necessary for a constant-angle-of-attack approach.

$\Delta C_m$ is also a function of the initial steady-state lift coefficient $C_{L0}$. Its absolute value usually increases with an increase in $C_{L0}$. In case of no significant change in $\Delta C_L/C_{L0}$, $\Delta\epsilon/C_{L0}$ and $\Delta a_t/a_{t0}$ due to variation of $C_{L0}$, it follows that $\Delta C_m$ is essentially a linear function of $C_{L0}$.

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# Fail-Safe Criteria and Analysis of VTOL Dynamic Component Structures

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## I. Introduction

PRESENT practice requires factors of safety such as 1.5 for static limit stresses and some reduction from the mean fatigue strength to insure that failure is remote from the operating loads environment. However, in addition to the mentioned factors of safety, greater emphasis is now being placed on the ability to operate safely under conditions where damage has been imparted to the structure. For example, the tentative[1] airworthiness standards of the Department of Transportation requires that an assessment of the residual fatigue strength after a partial failure must be made, and furthermore, permits an analytical assessment to be acceptable where such methods are shown to be reliable. Alternately, the VTOL manufacturer will be faced with test verification for each of the structural components deemed a safety of flight item.

## II. Fail-Safe Design Criteria

The design criteria for VTOL aircraft must encompass both the static residual strength and the crack propagation

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